PROCEEDINGS
OF
THE ROYAL
SOCIETY B

Proc. R. Soc. B (2005) **272**, 267–275
doi:10.1098/rspb.2004.2942
Published online 04 February 2005

Character complexity and redundancy in writing systems over human history

Mark A. Changizi[1*] **and Shinsuke Shimojo**[2,3]

[1]*Sloan-Swartz Center for Theoretical Neurobiology, and* [2]*Division of Biology, Computation and Neural Systems, MC 139-74,*
Caltech, Pasadena, CA 91125, USA
[3]*NTT Communication Science Laboratory, Atsugi, Kanagawa, Japan*

A writing system is a visual notation system wherein a repertoire of marks, or strokes, is used to build a repertoire of characters. Are there any commonalities across writing systems concerning the rules governing how strokes combine into characters; commonalities that might help us identify selection pressures on the development of written language? In an effort to answer this question we examined how strokes combine to make characters in more than 100 writing systems over human history, ranging from about 10 to 200 characters, and including numerals, abjads, abugidas, alphabets and syllabaries from five major taxa: Ancient Near-Eastern, European, Middle Eastern, South Asian, Southeast Asian. We discovered underlying similarities in two fundamental respects.

(i) The number of strokes per characters is approximately three, independent of the number of characters in the writing system; numeral systems are the exception, having on average only two strokes per character.
(ii) Characters are *ca*. 50% redundant, independent of writing system size; intuitively, this means that a character's identity can be determined even when half of its strokes are removed.

Because writing systems are under selective pressure to have characters that are easy for the visual system to recognize and for the motor system to write, these fundamental commonalities may be a fingerprint of mechanisms underlying the visuo–motor system.

Keywords: writing; reading; redundancy; complexity; letter perception; character recognition

1. INTRODUCTION

Writing systems (such as alphabets) are visual notation systems wherein a repertoire of marks, or strokes, is used to construct a set of characters. Because writing systems are under selective pressure to be easy to read and write, we reasoned that by identifying commonalities underlying how strokes combine into characters in writing systems over human history, we would, in effect, be identifying fundamental properties of the human visuo–motor system. Here, we are interested in measuring two specific fundamental properties of writing systems: *character length*, which is the average number of strokes per character, and *redundancy*, which measures how efficiently characters are built out of strokes. Our main result will be that, after examining more than 100 writing systems over human history (table 1), ranging from about 10 to 200 characters, we determined that writing systems have average lengths of approximately three strokes per character and redundancies of *ca*. 50%, and these values do not vary much as a function of writing system size. In § 4 we will speculate on what this might tell us about the human visuo–motor system.

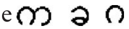
2. RESULTS

Suppose a writing system possesses B stroke types, and average character length L. We can model the number of characters, C, by an equation of the general form, $C = \sigma B^{\beta L}$, where $\sigma, \beta \leqslant 1$ are positive constants. The proportionality constant, σ, captures the fact that some fraction of the possible stroke combinations may not be allowed. The exponent βL is called the *combinatorial degree*, d, and measures how combinatorially strokes are used to build characters: a minimum value of $d = 1$ would mean that strokes are not used combinatorially at all (i.e. because doubling the number of stroke types would only double the number of characters), and greater values (up to a maximum of L) mean that strokes are used more combinatorially. From the length, L, and combinatorial degree, d, we may compute the *redundancy*, $R = 1 - (d/L)$, or $R = 1 - \beta$: a redundancy of zero means that all L potential degrees of freedom in building characters are used, and as the redundancy nears 1 the combinatorial degree falls lower and lower below L. For example, suppose that 0s and 1s can be strung together to make sequences of length 4, but that 0s must always be placed next to a 0, and 1s next to a 1. Although there are $2^4 = 16$ binary sequences of length 4, there are only $C = 4$ sequences satisfying this constraint, namely 0000, 0011, 1100 and 1111. Here, $\sigma = 1$ and $\beta = 1/2$, so that $C = 4 = 1 \times 2^{(1/2)4} = \sigma B^{\beta L}$. Intuitively, the constraints put on these sequences reduces the number of degrees of freedom from 4 down to 2, and thus the sequence length is twice as long as it ideally would have to be, corresponding to a redundancy of 1/2. The three properties—combinatorial degree d, length L, and redundancy R—are related such that any two are independent,

[*] Author for correspondence (changizi@changizi.com).

Received 9 August 2004
Accepted 16 September 2004

Table 1. Information for the writing systems used in the analysis.
(Character information was sampled from Ager's *Omniglot: a guide to writing systems* (Ager 1998), in conjunction with Daniels & Bright (1996). Writing systems were chosen from *Omniglot* so as to cover all major phylogenetically distinct writing system classes for numerals, abjads, abugidas (including vowel diacritics), alphabets and syllabaries, as shown in figure 1. Invented writing systems were included if they have been successfully used by some community; writing systems invented for fun or for fictional purposes were not included. We did not include logographic writing systems like Chinese and other East Asian writing systems where the character and word levels are not cleanly separable (see Chen *et al.* 1996; Yeh & Li 2004). When multiple variants existed, we chose the first shown. Lower-case characters were used unless otherwise noted; also, character variants that occur at the start or end of a word were not used. In several alphabets, the alphabet has some syllabic features, and 'syllabic' has been placed in parentheses to denote this. Dates are often only approximate, and are recorded such that negative values indicate BC. The 'phylogeny' column shows the principal sequence of ancestors of the writing system; or when invented, it gives the inventor's name. The fifth column gives the average character length, which is the average number of strokes per character in the writing system (see figure 2*a*). The sixth column gives the stroke type repertoire size (see figure 3*a*), and the seventh column the writing system size (i.e. the number of characters). The last column provides the total number of edges in the stroke-type network for that writing system (see text and figure 4*a* for more information on stroke-type networks).)

	name and sample characters	kind of system	date	phylogeny	average character length L	number of stroke types B	number of characters C	total number of edges
1	Ahom	abugida	1250	Brahmi →	2.40	26	40	68
2	Albanian (Elbasan)	alphabet	1750	invented: unknown	2.60	32	53	82
3	Ancient Berber (Vertical)	abjad	−150	Punic →	2.68	12	25	23
4	Arabic	abjad	512	Aramaic → Nabataean →	2.63	19	35	36
5	Arabic	numeral	512	Aramaic → Nabataean Aramaic →	2.00	10	10	13
6	Aramaic	abjad	−900	Phoenician →	2.68	11	22	25
7	Armenian (Eastern)	alphabet	405	invented: Mesrop-Mashtots	2.21	24	39	42
8	Asomtavruli	alphabet	430	Greek →	2.42	25	38	52
9	Avestan	alphabet	250	Aramaic → Psalter and Old Pahlavi →	2.42	35	53	77
10	Bassa	alphabet	unknown	invented: unknown	2.37	24	30	38
11	Batak (Kara Batak)	abugida	1350	Brahmi → Pallava → Old Kawi →	2.15	13	33	33
12	Bengali	abugida	1050	Brahmi → Devanagari →	3.91	29	45	113
13	Bengali	numeral	1050	Brahmi → Devanagari →	1.80	14	10	15
14	Brahmi	abugida	−450	Aramaic →	2.14	24	43	51
15	Buhid (Mangyan)	abugida	1350	Brahmi → Pallava → Old Kawi →	3.78	5	18	12
16	Burmese 	abugida	1150	Brahmi → Mon →	2.51	29	41	85
17	Burmese	numeral	1150	Brahmi → Mon →	1.40	10	10	6
18	Carrier (Dene)	syllabary (rotating)	1885	invented: Father Adrien-Gabriel Morice	3.20	61	180	68
19	Celtiberian	alphabet (syllabic)	−550	Punic → Iberian →	3.29	9	28	36
20	Cherokee	syllabary	1819	invented: George Guess	2.35	51	85	102
21	Chinese	numeral	−1150	unknown	3.00	6	10	13
22	Cypriot	syllabary	−800	linear B →	3.84	19	55	68
23	Cyrillic (Abkhaz)	alphabet	950	Greek → Old Church Slavonic →	3.69	19	62	42
24	Dehong	abugida	1050	Brahmi → Sinhala →	3.39	13	33	40
25	Dehong	numeral	1050	Brahmi → Sinhala →	1.90	11	10	12
26	Deseret	alphabet	1850	invented: George D. Watt	1.68	27	38	34
27	Devanagari	abugida	1050	Brahmi →	3.27	30	45	83
28	Devanagari	numeral	1050	Brahmi →	1.40	13	10	8
29	Dives Akuru	abugida	1200	Brahmi → Sinhala →	3.77	21	23	31
30	Enochian	alphabet	1580	invented: Dr John Dee and Sir Edward Kelly	2.52	19	21	37
31	Ethiopic (Ge'ez)	abugida	350	Southern Linear → Sabaean/Minean →	2.63	20	40	41

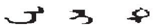
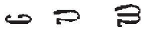
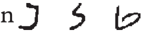
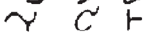
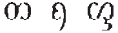
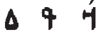
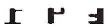
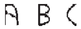
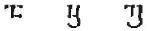
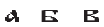
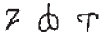
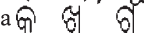
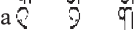
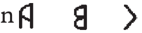
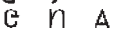
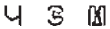
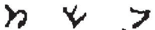
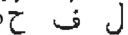
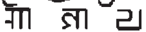
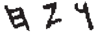
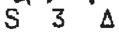
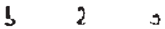
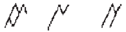
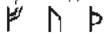
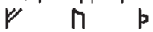
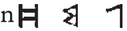
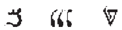
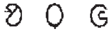
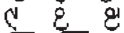
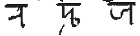
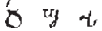
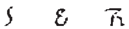
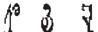
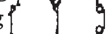
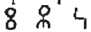
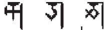
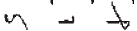
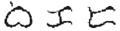
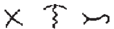
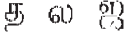
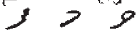
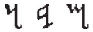
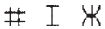
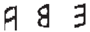
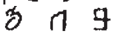
(*Continued.*)

Table 1. (*Continued.*)

	name, and sample characters	kind of system	date	phylogeny	average character length L	number of stroke types B	number of characters C	total number of edges
32	Etruscan (archaic)	alphabet	−750	Greek →	2.91	11	23	25
33	Faliscan	alphabet	−650	Greek → Etruscan →	2.52	15	21	32
34	Fraser	alphabet	1915	invented: James Ostram Fraser	2.37	17	41	36
35	Glagolitic	alphabet	860	Greek →	4.51	22	41	95
36	Gothic (Wulfila)	alphabet	350	Greek →	2.32	17	25	37
37	Greek α β γ	alphabet	−750	Phoenician →	1.71	21	24	24
38	Gujarati	abugida	1592	Brahmi → Devanagari →	2.21	32	42	63
39	Gujarati	numeral	1592	Brahmi → Devanagari →	1.50	13	10	10
40	Gurmukhi	abugida	1550	Brahmi →	3.20	31	46	93
41	Gurmukhi	numeral	1550	Brahmi →	1.90	12	10	11
42	Hanuno'o (Mangyan)	abugida	1350	Brahmi → Pallava → Old Kawi →	3.13	13	16	34
43	Hebrew	abjad	−125	Aramaic →	2.55	18	33	30
44	Hindu-Arabic 1 2 3	numeral	700	Brahmi →	1.60	9	10	9
45	Hungarian Runes	alphabet	1000	Aramaic → Sogdian → Turkic →	3.08	12	40	34
46	Hungarian Runes | || |||	numeral	1000	Aramaic → Sogdian → Turkic →	2.50	4	6	12
47	Iberian (northern)	alphabet (syllabic)	−1000	Punic →	3.46	11	26	27
48	Iberian (southern)	alphabet (syllabic)	−1000	Punic →	3.14	11	22	30
49	International phonetic	alphabet	1847	invented: Isaac Pitman and Henry Ellis	2.42	47	170	119
50	Kannada	abugida	550	Brahmi →	2.79	34	47	92
51	Kannada	numeral	550	Brahmi →	1.00	10	10	0
52	Kharoshthi	abugida	−450	Aramaic →	1.72	30	39	44
53	Kharoshthi	numeral	−450	Aramaic →	1.88	7	8	11
54	Khmer	abugida	611	Brahmi → Pallava →	7.49	33	68	70
55	Khmer	numeral	611	Brahmi → Pallava →	3.70	14	10	30
56	Korean (Hangeul)	alphabet	1446	invented: King Seycong	2.83	8	24	19
57	Kpelle	syllabary	1930	invented: Chief Gbili	3.07	74	88	198
58	Latin, ancient	alphabet	−650	Greek → Etruscan →	2.67	10	21	25
59	Latin, modern a b c	alphabet	1600	Greek → Etruscan → ancient Latin →	2.08	14	26	33
60	Latin, modern all-caps A B C	alphabet	1600	Greek → Etruscan → ancient Latin →	2.50	17	26	41
61	Lepcha (Rong)	abugida	1720	Brahmi → Devanagari → Tibetan →	2.68	44	77	95
62	Lepcha (Rong)	numeral	1720	Brahmi → Devanagari → Tibetan →	2.60	15	10	27
63	Limbu	abugida	1730	Brahmi → Devanagari → Tibetan → Lepcha →	2.51	34	37	72
64	Linear B	syllabary	−1550	linear A →	5.03	34	73	148
65	Marsiliana	alphabet	−650	Greek →	2.88	15	26	33
66	Meroitic (non-hieroglyphic)	abugida	−250	Ancient Egyptian →	3.46	19	23	55
67	Messapic	alphabet	−550	Greek →	2.87	14	23	34
68	Middle Adriatic (South Picene)	alphabet	−650	Greek → Etruscan →	2.70	13	23	32
69	Middle Persian (Pahlavi)	abjad	200	Aramaic →	2.00	16	22	25
70	Mkhedruli	alphabet	1200	Greek → Asomtavruli → Nushka-khucuri →	2.21	38	38	73

(*Continued.*)

Table 1. (*Continued*.)

	name, and sample characters	kind of system	date	phylogeny	average character length *L*	number of stroke types *B*	number of characters *C*	total number of edges
71	Mongolian 	alphabet	1150	Aramaic → Sogdian →	3.29	28	35	77
72	Mongolian 	numeral	1150	Aramaic → Sogdian →	1.60	11	10	9
73	Nabataean 	abjad	50	Aramaic →	1.77	18	22	24
74	Ndjuka' 	syllabary	1910	invented: Afaka Atumisi	2.35	36	52	78
75	New Tai Lue 	numeral	1950	invented: unknown	2.23	12	10	12
76	N'Ko 	alphabet	1949	invented: Soulemayne Kante	2.14	20	22	28
77	N'Ko 	numeral	1949	invented: Soulemayne Kante	2.60	6	10	9
78	North Picene 	alphabet	−650	Greek → Etruscan →	2.67	13	18	27
79	Nuskha-khucuri 	alphabet	850	Greek → Asomtavruli →	3.58	16	38	31
80	Old Church Slavonic 	alphabet	850	Greek →	3.33	27	42	70
81	Old Permic (Abur) 	alphabet	1390	invented: St. Stephen of Perm	3.11	29	38	71
82	Oriya 	abugida	1051	Brahmi → Kalinga →	2.89	34	44	101
83	Oriya 	numeral	1051	Brahmi → Kalinga →	1.50	15	10	10
84	Oscan 	alphabet	−650	Greek → Etruscan →	2.71	14	21	28
85	Pahawh Hmong 	alphabet (unusual)	1959	invented: Shong Lue Yang	3.30	42	86	95
86	Pahawh Hmong 	numeral	1959	invented: Shong Lue Yang	2.50	15	10	30
87	Parthian 	abjad	100	Aramaic →	2.59	14	22	25
88	Pashto 	abjad	600	Aramaic → Nabataean → Arabic →	2.73	20	40	47
89	'Phags-pa 	abugida	1269	Brahmi → Devanagari → Tibetan →	4.53	21	40	56
90	Phoenician 	abjad	−1250	northern linear (Canaanite) →	2.86	13	22	34
91	Pollard Miao 	alphabet (unusual)	1905	invented: Samuel Pollard	2.11	22	47	32
92	Psalter 	abjad	100	Aramaic →	2.00	19	21	28
93	Redjang (Kaganga) 	abugida	1350	Brahmi → Pallava → Old Kawi →	3.00	10	36	16
94	Runic (Danish Futhark) 	alphabet	800	unknown	2.63	10	16	21
95	Runic (Elder Futhark) 	alphabet	50	unknown	3.13	6	24	16
96	Sabaean/Minean 	abjad	−500	southern linear → Sabaean/Minean →	3.55	12	29	28
97	Samaritan 	abjad	−50	Aramaic → Old Hebrew ??? →	4.32	24	22	72
98	Santali (Ol Cemet') 	alphabet	1920	invented: Pandit Raghunath Murmu	3.33	29	30	80
99	Santali (Ol Cemet') 	numeral	1920	invented: Pandit Raghunath Murmu	1.40	11	10	7
100	Sil'oti Nagri 	abugida	1300	invented: Saint Shahjalal	3.33	28	34	69
101	Somali (Osmanya) 	alphabet	1922	invented: Cismaan Kenadiid	1.90	29	30	38
102	Somali (Osmanya) 	numeral	1922	invented: Cismaan Kenadiid	1.50	15	10	10
103	Sorang Sompeng 	alphabet	1936	invented: Mangei Gomango	2.29	34	24	58
104	Sorang Sompeng 	numeral	1936	invented: Mangei Gomango	1.40	12	10	8
105	South Arabian 	abjad	−600	southern linear →	3.29	13	28	36
106	Soyombo 	abugida	1686	invented: Bogdo Zanabazar	3.63	27	35	88
107	Syriac 	abjad	400	Aramaic →	2.27	25	22	50
108	Tagalog 	abugida	900	Brahmi → Pallava → Old Kawi →	1.93	17	16	23
109	Tagbanwa 	abugida	900	Brahmi → Pallava → Old Kawi →	2.23	19	15	26
110	Tamil 	abugida	−300	Brahmi →	2.74	29	34	62
111	Thaana 	abugida	1550	invented: Unknown	2.09	23	35	42
112	Theban 	alphabet	100	invented: Unknown	3.83	28	24	69
113	Tifinagh 	abjad	−100	Punic → Ancient Berber →	2.88	13	25	23
114	Umbrian 	alphabet	−350	Greek → Etruscan →	2.48	16	21	30
115	Varang Kshiti 	alphabet	1900	invented: Lako Bodra	2.86	17	21	26

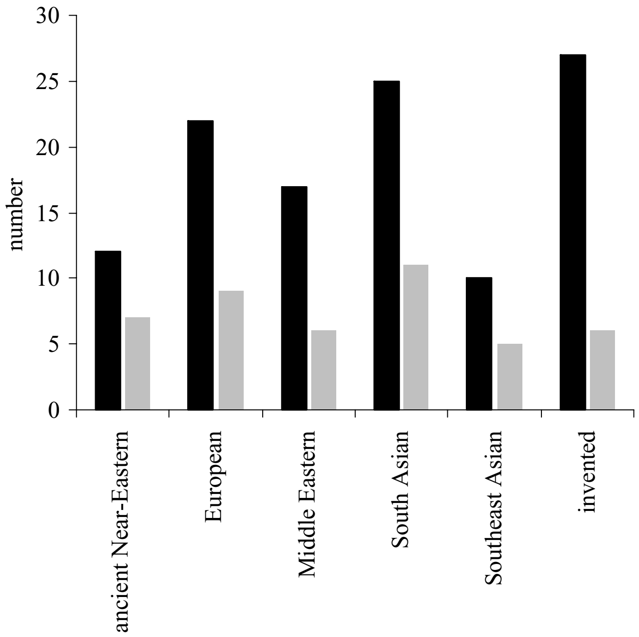


Figure 1. Distribution of writing systems used in the study (see table 1) across the major phylogenetic classes (black bars), and also the number of sections devoted to the phylogenetic classes in Daniels & Bright (1996) *The world's writing systems*, the most exhaustive book on the topic (grey bars). Among the major phylogenetic classes, the distributions are highly correlated ($r^2 = 0.81$).

and jointly determine the third via the equation $R = 1 - (d/L)$. There are two qualitatively different ways that strokes could be combinatorially used to build characters (Changizi 2001, 2003*a*,*b*). The first is the *universal stroke-type approach*, where the number of stroke types does not vary as a function of writing system size, and greater numbers of characters are accommodated by increasing the length of characters. The second is the *invariant-length approach*, where character length is invariant, and greater numbers of characters are accommodated by increasing the number of stroke types from which characters are built.

To test whether either of these two scaling approaches applies to writing systems, we measured average character lengths for all 115 writing systems in table 1 (see figure 1 for the distribution of classes of writing system). Figure 2*a* illustrates the manner in which characters are decomposed into strokes. Our first result is that writing systems appear to conform to the invariant-length approach (see figure 2*b*), with average lengths of approximately 3 (but with lower lengths of approximately 2 for number systems).

For the remainder of § 2, we describe two distinct methods for estimating the combinatorial degree, $d = \beta L$. From the length, L, and combinatorial degree, d, we will be able to compute the redundancy, $R = 1 - (d/L)$.

The first method of estimating combinatorial degree is to plot stroke-type repertoire size, B, as a function of writing system size, C, and measure the scaling exponent. Recall that $C \propto B^d$. Thus, $B \propto C^{1/d}$, and the best-fit slope on a log-log plot of B versus C is an estimate of $1/d$. Figure 3*a* illustrates the manner in which the stroke-type repertoire is determined, and figure 3*b* describes tests of repeatability. We let d_{BC} denote estimates of the combinatorial degree via this first method. We report four estimates from the data, employing four different subscripts to distinguish between them—'all', 'alpha', 'all,bin' and 'alpha,bin'—where the subscript 'all' means that all writing systems from table 1 are used in the estimate, 'alpha' means that only the non-number systems are used, and 'bin' means that the estimate is taken from a binned plot. Figure 3*c* shows the plot of all the data, and $B \propto C^{0.63}$, and thus $d_{BC,\text{all}} = 1/0.63 = 1.60$. The inset of figure 3*c* shows the binned version of all the data, and $d_{BC,\text{all,bin}} = 1.49$. Excluding numerals, the respective estimates are $d_{BC,\text{alpha}} = 1.36$ and $d_{BC,\text{alpha,bin}} = 1.33$. These combinatorial degree estimates therefore range from 1.33 to 1.60. Given the average character lengths, the four corresponding redundancy estimates are: $R_{BC,\text{all}} = 41\%$, $R_{BC,\text{all,bin}} = 46\%$, $R_{BC,\text{alpha}} = 53\%$ and $R_{BC,\text{alpha,bin}} = 56\%$.

The second method for determining the combinatorial degree is via measuring how stroke-type 'interactiveness' changes with writing system size. We let d_{deg} denote estimates of the combinatorial degree via this second method. If strokes are used combinatorially, then in a larger writing system, any given stroke type must, on average, be able to interact with a greater number of stroke types. The *degree*, δ, of a stroke type is the total number of stroke-types with which the stroke type intersects, across all characters of the writing system (for cases of unconnected strokes, like the dot of an 'i', the stroke was deemed connected to the nearest stroke). Figure 4*a* illustrates the manner in which the stroke-type degree is determined. Figure 4*b* shows that the average stroke-type degree changes slowly with writing system size, consistent with a power law $\delta \propto C^w$: Scaling exponents via the four methods are $w_{\text{all}} = 0.24$, $w_{\text{all,bin}} = 0.17$, $w_{\text{alpha}} = 0.13$, $w_{\text{alpha,bin}}$=0.10. From this it is possible to compute the combinatorial degree, as we now explain. How many characters can be built with length L? In writing a single character, there are B stroke types one may begin with, and for each of these there are, on average, δ many stroke types which may be drawn next, and for each of these, δ more, and so on until all L strokes have occurred in the character. Thus, the number of characters that can be built is $C = B\delta^{L-1}$. Given that $\delta \propto C^w$, we can write $C \propto B(C^w)^{L-1}$, and solving for C, we have that $C \propto B^{1/[1-w(L-1)]}$. Therefore, the combinatorial degree can be estimated as $d_{\text{deg}} = 1/[1 - w(L - 1)]$, where w is the scaling exponent for stroke-type degree as a function of writing system size (see Changizi *et al.* (2002) for related observations). The four estimates of combinatorial degree using stroke-type degree scaling are $d_{\text{deg,all}} = 1.73$, $d_{\text{deg,all,bin}} = 1.42$, $d_{\text{deg,alpha}} = 1.32$ and $d_{\text{deg,alpha,bin}} = 1.26$. These combinatorial degree estimates therefore range from 1.26 to 1.73, similar to the range of 1.33 to 1.60 that we found earlier via the first method. The corresponding redundancies are $R_{\text{deg,all}} = 36\%$, $R_{\text{deg,all,bin}} = 49\%$, $R_{\text{deg,alpha}} = 54\%$ and $R_{\text{deg,alpha,bin}} = 58\%$.

3. DISCUSSION

We found that writing systems have average character lengths of approximately 3 (number systems being the exception, with an average of approximately 2). And, via two distinct kinds of measurement and analysis, we found that the combinatorial degrees for writing systems are very approximately 3/2, and redundancies *ca.* 50%. Importantly, these values appear to not much vary as a function of writing system size. Because the combinatorial degree is

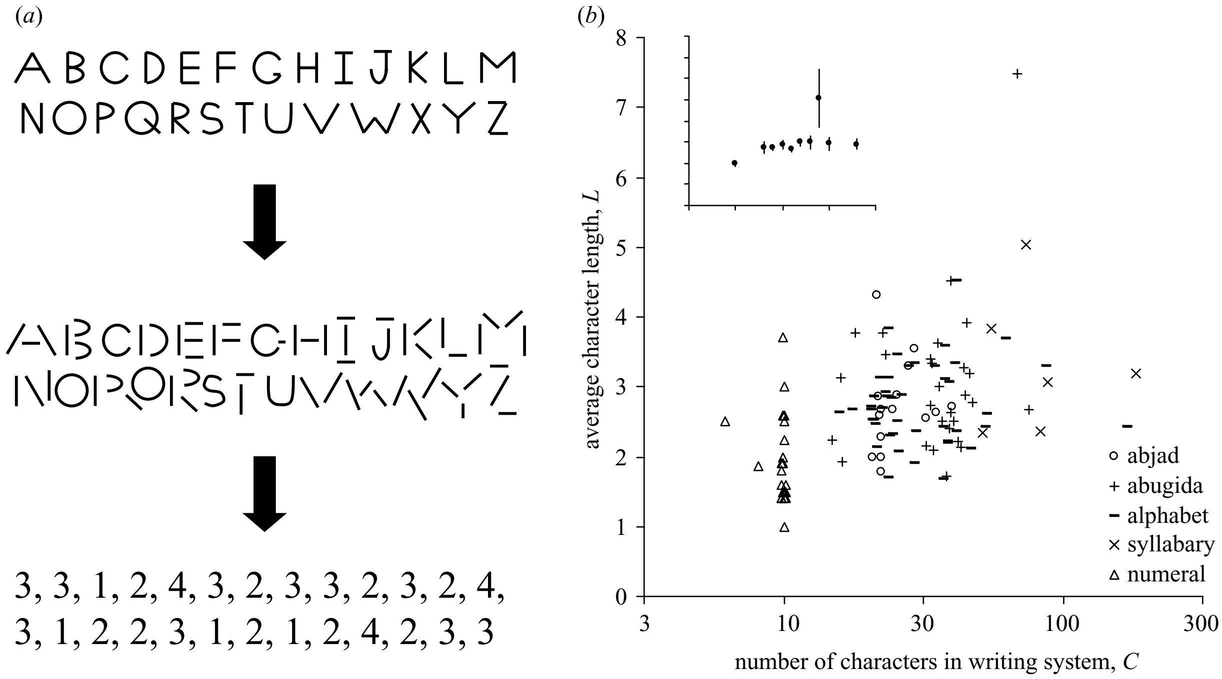


Figure 2. (*a*) Illustration of the method for determining character lengths (i.e. the number of strokes per character). Each character is decomposed into separable strokes, where strokes are separated by discontinuities so that ‘U’ is one stroke but ‘V’ is two, and also stroke junctions are decomposed into their constituents so that ‘T’ and ‘X’ junctions possess two strokes, ‘Y’, ‘K’ and ‘Ψ‘ junctions possess three strokes, etc. Three naïve observers were asked to decompose characters into strokes, and there were no disagreements. (*b*) Plot of average character length versus the number of characters (on a log scale), for 115 writing systems. Data are labelled by abjad (characters for consonants but not vowels), abugidas (characters for consonants and diacritic symbols for vowels), alphabets (characters for consonants and vowels), syllabaries (characters for syllables such as ‘ba’, ‘be’, ‘bi’, etc.) and numerals (characters for numbers). *x*-axis values have been randomly perturbed by $\pm 1\%$ to help distinguish the points on the plot. The average length is 2.79 for invented systems (a set of 38 *independent* writing systems) and 2.70 for non-invented systems. Inset: plot of the same data, and same axes, but average character lengths binned at 0.1 intervals along the *x*-axis (standard error bars shown). One can see that, except for number systems where the average length is approximately 2 (the average across the average lengths of the 22 numeral systems is 1.95, with standard error 0.14), the average character length does not appear to vary as a function of writing system size (the average across the average lengths of the 93 non-numeral systems is 2.91, with standard error 0.09). These data mean that human writing systems conform to the invariant-length approach to accommodating writing systems of greater size.

significantly above 1, it means that writing systems use strokes in a genuinely combinatorial fashion (i.e. doubling the number of strokes *more* than doubles the number of characters, because $C \propto B^{3/2}$). However, although writing systems are combinatorial, they are not very combinatorial, because the combinatorial degree of 3/2 is not much greater than 1. Because average character lengths are approximately 3, the maximum possible combinatorial degree is 3. Because only approximately half of the total possible degrees of freedom is used, the redundancy is *ca*. 50%. Characters therefore tend to be about twice as long (in number of strokes) as they need to be. Alternatively, the combinatorial degree could be twice what it is, which would allow the number of stroke types to grow much more slowly as a function of writing system size (namely as the cube root) than they in fact do. These results may have implications for the future design of writing systems.

4. CONCLUSION

Writing systems are under selective pressure to be easy to read and write, but there are reasons to think that the principal pressure is for ease of reading. First, text is written only once, whereas it may be read arbitrarily many times. The utilities due to reading will accordingly be amplified relative to that for writing. Many writing systems throughout history, however, were not read to the extent that contemporary writing systems are, and this argument will not apply as strongly to such writing systems. Second, cursive scripts and shorthand are two classes of writing system where selection is primarily driven by writing optimization, and in these cases the characters are qualitatively very different compared with those of the typical writing system, and are more difficult to read. Third, and last, typeface and computer fonts are two classes of script where there is no selective pressure for writing at all, and characters in these scripts are qualitatively quite similar to those of the typical writing system. None of these above arguments alone is strong, but together they give us some reason to suspect that the principal selective pressure on most writing systems may come from vision. Assuming this, we ask, is there something about these fundamental properties of writing systems that might be ‘good’ for the visual system?

Consider redundancy first. Because character recognition requires recognizing the strokes (Pelli *et al.* 2004), and because strokes tend have small angular size and high shape variability, some redundancy is useful so that misrecognition of one or two strokes does not necessarily lead to misrecognition of the character. Why should the visual

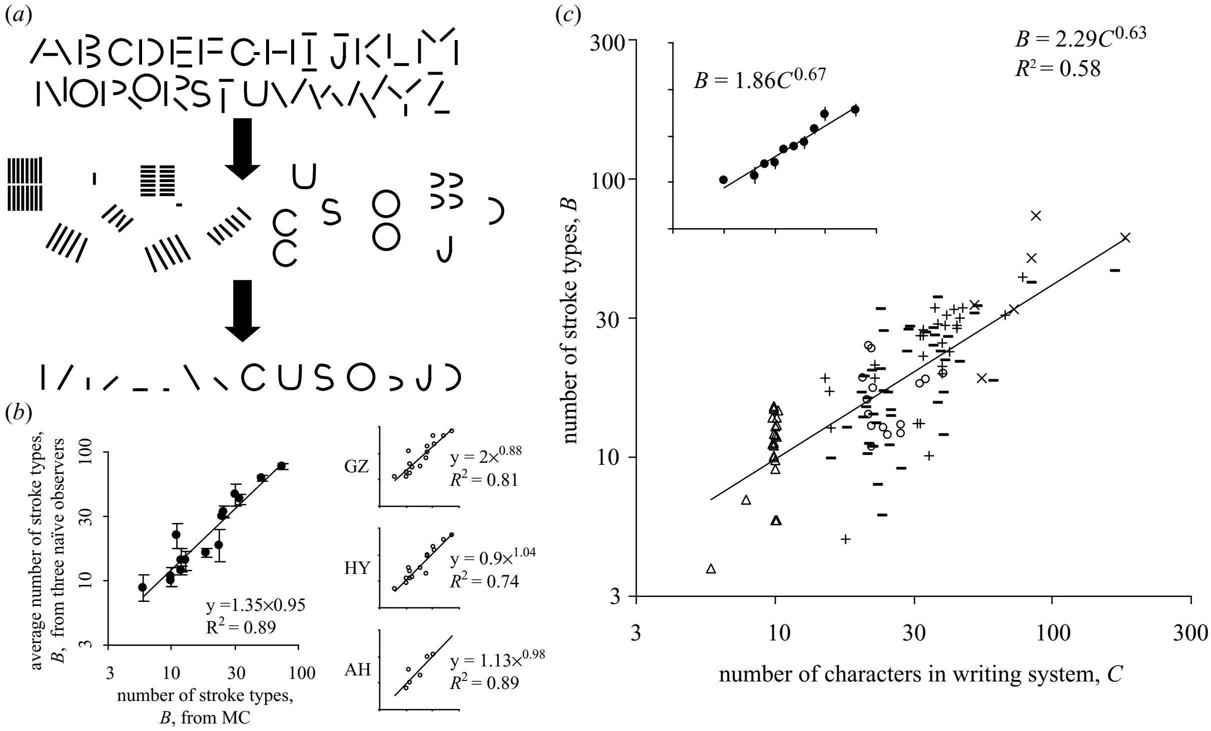


Figure 3. (*a*) Illustration of how the stroke-type repertoire is determined for a writing system. After the characters are decomposed into their constituent strokes (see figure 1*a*), the strokes are clustered near strokes that appear to be similar. Stroke types were determined by the primary author (M.C.) on the basis of high intra-cluster similarity in orientation, shape and length. (*b*) As a test of repeatability, three naïve observers (G.Z., H.Y. and A.H.) were asked to determine the stroke-type repertoire for a wide variety of writing systems (G.Z. and H.Y. carried this out for Ancient Berber, Ahom, Albanian, Arabic, Arabic numerals, Aramaic, Armenian, Asomtavruli, Avestan, Hanuno'o, Cherokee, Hungarian Runes, Elder Futhark, Danish Futhark, Kpelle; and A.H. carried this out for just the first six). On the left is a log–log plot of the average stroke-type repertoire size measured by the three naïve observers versus the estimates of M.C. Standard error bars are shown, as well as the best-fit (by linear regression) equation and line, and the correlation. One can see that the correlation is high and that the exponent relating them is approximately 1, meaning that naïve observers' estimates of stroke-type repertoire size scale in direct proportion to the estimates of M.C. The three plots on the right possess the same *x*-axis as the one on the left, but the *y*-axis now has each individual observer's stroke-type estimates. The effects of systematic under- or over-counting (as seen for example in G.Z.) will affect the proportionality constant relating stroke-type repertoire size, *B*, to writing system size, *C*, but not the scaling exponent, which is what is of interest to us here. (*c*) Plot of number of stroke types versus number of characters for 115 writing systems. Circles, abjad; plus symbols, abugida; minus symbols, alphabet; crosses, syllabary; and triangles, numerical. The linear regression line and equation are shown, along with correlation. Data points on each axis have been perturbed by ±1% to aid in their discrimination. The best-fit relationship is $B = 3.18C^{0.57}$ for invented systems (a set of independent data), and $B = 2.31C^{0.60}$ for non-invented systems. Inset: same plot, and same axes, but stroke-type repertoire sizes binned at 0.1 intervals along the log *C*-axis (standard error bars shown).

system prefer character lengths of approximately 3? The value of 3 naturally suggests the 'subitizing limit', which is the number of objects that can be stored in visual short term memory, and is often put at roughly 3 (e.g. Trick & Pylyshyn 1994; Vogel *et al.* 2001). That is, perhaps there are, on average, three strokes per character, independent of writing system size, because all the strokes can be simultaneously processed, whereas processing times increase substantially for greater than around three objects. It has been thought that this may underly why number systems tend to represent '1' by one stroke, '2' by two strokes, and '3' by three strokes, but this stops for greater numbers (Ifrah 1985; Zhang & Norman 1995; Dehaene 1997). The combinatorial degree value of 3/2, and the connected rate at which the number of stroke types increases with writing system size (namely as the 3/2 power), would be a consequence of the redundancy and subitizing limit.

A distinct possible kind of explanation for the average length of 3 concerns bottom-up, hierarchical processing of characters (A. Hampton, private communication). Imagine a lower-level retinotopic map in visual cortex, where the *L* strokes of a character are simultaneously recognized in *L* nearby regions of the cortex. Suppose also that multiple nearby regions in the lower level connect in a feed-forward fashion to a single region of the upper-level retinotopic area. A single upper-level region could integrate only from as many lower-level regions as connect to it, and perhaps character length *L* would be constrained by this. Supposing that multiple lower-level regions feed-forward to an upper-level region if and only if the lower-level regions are all mutually adjacent, and assuming that regions are hexagonally packed in neocortex, each upper-level region will integrate exactly three mutually adjacent lower-level regions, which would cohere with the average length of $L \approx 3$.

Another intriguing possibility is that there is a fundamental ecological explanation for these writing system features. The visual system has been selected to quickly

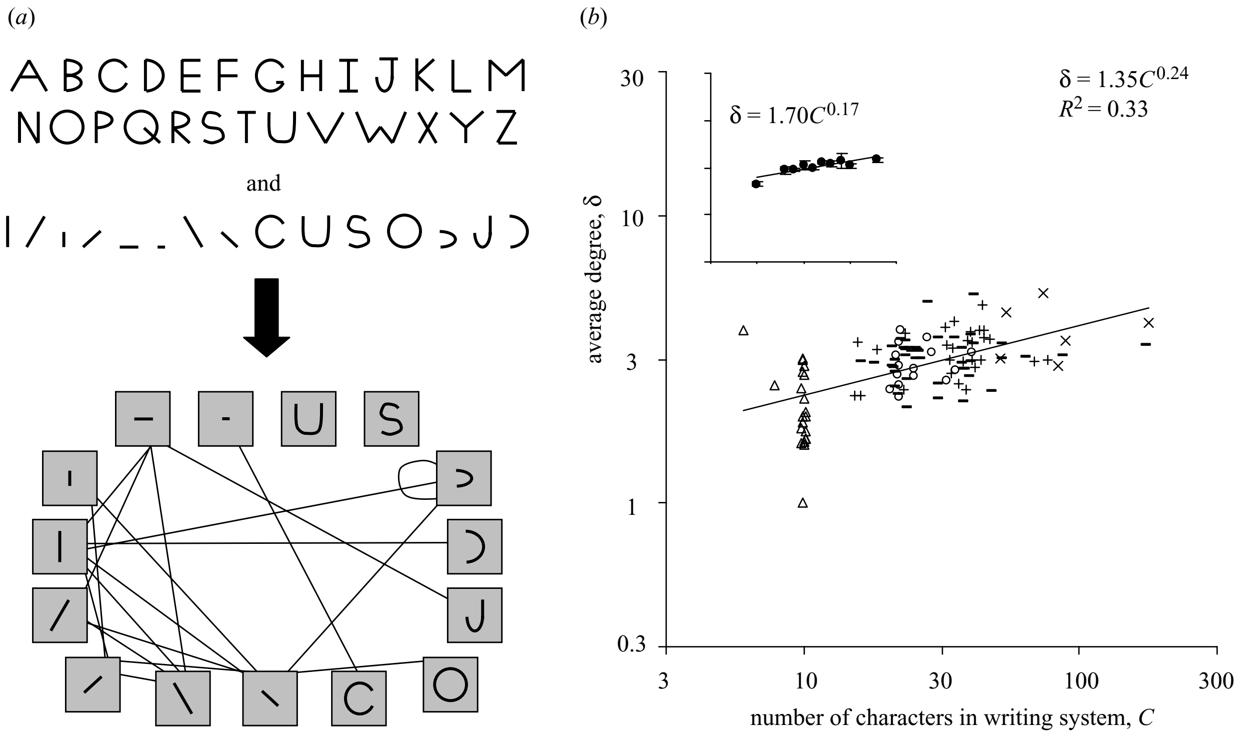


Figure 4. (*a*) Illustration of how a stroke-type network is built from the character repertoire and stroke type repertoire. Each stroke type is represented as a node in the network, and two stroke types are connected just in cases where those stroke types intersect in some character of the writing system; intuitively, stroke types sharing an edge in the network have the ability to 'interact'. When a stroke does not intersect other strokes of a character—like the dot of an 'i'—the stroke is deemed to intersect the physically nearest stroke. (*b*) Log–log plot of average stroke-type degree versus number of characters for 115 writing systems. Circles, abjad; plus symbols, abugida; minus symbols, alphabet; crosses, syllabary; and triangles, numerical. The linear regression line and equation are shown, along with correlation. x-axis values have been perturbed by ±1% to aid in their discrimination. The best-fit relationship is $\delta = 1.40C^{0.22}$ for invented systems (a set of independent data), and $\delta = 1.16C^{0.30}$ for non-invented systems. Inset: same plot, and same axes, but stroke-type degrees binned at 0.1 intervals along the log C-axis (standard error bars shown).

recognize objects, and many objects are built from object junctions of various kinds, which are themselves built out of contours of various types. Could it be that writing systems have been selected to have characters that can be recognized using the already-existing object recognition mechanisms? Intuitively, strokes are contour-like, and characters are object-junction-like, object junctions typically possessing approximately three intersecting contours (e.g. Clowes 1971; Huffman 1971; Chakravarty 1979). Might it be more than a coincidence that object junctions are well described as 'T', 'L', 'X', 'Ψ', 'K' and 'Y' junctions? A test of this hypothesis is the subject of ongoing research (Changizi *et al*. 2004).

We thank Dan Ryder, Wei Ji Ma, Patrick Wilken, Alan Hampton and two reviewers for their comments. We also thank Andrew Hsieh, Hao Ye and Qiong Zhang for independently parsing into stroke types a variety of writing systems.

REFERENCES

Ager, S. 1998 Omniglot: a guide to writing systems. See http://www.omniglot.com.

Chakravarty, I. 1979 A generalized line and junction labeling scheme with applications to scene analysis. *IEEE Trans. Pattern Analysis Machine Intell.* **1**, 202–205.

Changizi, M. A. 2001 Universal scaling laws for hierarchical complexity in languages, organisms, behaviors and other combinatorial systems. *J. Theor. Biol.* **211**, 277–295.

Changizi, M. A. 2003*a* The relationship between number of muscles, behavioral repertoire, and encephalization in mammals. *J. Theor. Biol.* **220**, 157–168.

Changizi, M. A. 2003*b* *The brain from 25 000 feet: high level explorations of brain complexity, perception, induction and vagueness*. Dordrecht, The Netherlands: Kluwer.

Changizi, M. A., McDannald, M. A. & Widders, D. 2002 Scaling of differentiation in networks: nervous systems, organisms, ant colonies, ecosystems, businesses, universities, cities, electronic circuits, and Legos. *J. Theor. Biol.* **218**, 215–237.

Changizi, M. A., Zhang, Q., Ye, H. & Shimojo, S. 2004 The structures of letters and symbols throughout human history are selected to match those found in objects in natural scenes. (Submitted.)

Chen, Y. P., Allport, D. A. & Marshall, J. C. 1996 What are the functional orthographic units in Chinese word recognition: the stroke or the stroke pattern? *Q. J. Exp. Psychol. A Hum. Exp. Psychol.* **49**, 1024–1043.

Clowes, M. B. 1971 On seeing things. *Artificial Intell.* **2**, 79–116.

Daniels, P. T. & Bright, B. 1996 *The world's writing systems*. New York: Oxford University Press.

Dehaene, S. 1997 *The number sense: how the mind creates mathematics*. Oxford University Press.

Huffman, D. A. 1971 Impossible objects as nonsense sentences. In *Machine intelligence*, vol. 6 (ed. B. Meltzer & D. Michie), pp. 295–323. New York: Elsevier.

Ifrah, G. 1985 *From one to zero: a universal history of numbers*. New York: Viking Press.

Pelli, D. G., Burns, C. W., Farell, B. & Moore, D. C. 2004 Identifying letters. *Vision Res.* (In the press.)

Trick, L. M. & Pylyshyn, Z. W. 1994 Why are small and large numbers enumerated differently? A limited-capacity pre-attentive stage in vision. *Psychol. Rev.* **101**, 80–102.

Vogel, E. K., Woodman, G. F. & Luck, S. J. 2001 Storage of features, conjunctions, and objects in visual working memory. *J. Exp. Pscyhol. Hum. Percept. Perform.* **27**, 92–114.

Yeh, S. L. & Li, J. L. 2004 Sublexical processing in visual recognition of Chinese characters: evidence from repetition blindness for subcharacter components. *Brain and Language* **88**, 47–53.

Zhang, J. & Norman, D. A. 1995 The representation of numbers. *Cognition* **57**, 271–295.

As this paper exceeds the maximum length normally permitted, the authors have agreed to contribute to production costs.