Behavior Research Methods, Instruments, & Computers
1987, 19 (4), 409-412

F ratios and quasi *F* ratios for fixed, mixed, and random model ANOVAs

ALLEN H. WOLACH
Illinois Institute of Technology, Chicago, Illinois

and

MAUREEN A. MCHALE
Northwestern State University of Louisiana
Natchitoches, Louisiana

Statistical packages usually calculate analyses of variance as if all factors are fixed as opposed to random factors. Many investigators are unaware that *F* ratios are not formed the same way for fixed and random factors. Calculations for analyses of variance with fixed factors, random factors, and mixed designs (some factors fixed and some factors random) are identical until *F* ratios are formed. The appropriate mean squares used in the numerator and denominator of a given *F* ratio can vary from fixed to mixed to random model analyses of variance. An investigator may perform an analysis of variance using his or her favorite statistical package (e.g., SPSS). After the summary table for the analysis of variance is printed, the investigator can use the degrees of freedom in the df column and the mean squares in the MS column of the summary table to form appropriate *F* ratios.

Kirk (1982, pp. 390-395) provided rules for determining numerators and denominators of *F* ratios. In order to apply these rules, an investigator must be able to write the experimental design model equation for the analysis of variance, set up a two-way table (design model terms by subscripts for terms), enter sampling fractions (e.g., $1-p/P$), assign values to the sampling fractions (0 for fixed factors and 1 for random factors), and so forth.

The present program determines which degrees of freedom and which mean squares are used for the numerator and denominator of each *F* ratio. The appropriate analysis of variance is selected from a list of analyses of variance that appears on the monitor screen. The user enters R (random) or F (fixed) for each factor. Then the program determines the appropriate terms for the numerators and denominators of each *F* ratio. Sometimes there is no appropriate single mean square for the numerator and single mean square for the denominator of an *F* ratio. The program determines the mean squares that should be combined to form quasi *F* ratios. The user has the option of choosing quasi *F* ratios formed by Cochran's (1951) or Satterthwaite's (1946) method. Kirk (1982) gives an excellent explanation of fixed, mixed, and random models and a discussion of the Cochran and Satterthwaite methods for forming quasi *F* ratios (pp. 390-395). Kirk also discusses fixed, mixed, and random models in terms of differences in sampling fractions for the three kinds of designs.

Input. The program provides a menu with analyses of variance that have from one to five factors. Any or all of the factors can be independent groups factors or repeated measures factors. That is, an investigator can choose an independent groups design with one or more factors, a split-plot design with two or more factors, a randomized block, or a randomized block factorial design. The program assumes that the earlier factors are the independent groups factors. For example, an SPF-JKL.M design is for a split-plot factorial analysis of variance with independent groups factors J, K, and L and a repeated measures factor M.

The program places a request on the monitor for an entry of F (fixed) or R (random) for each of the factors in the analysis of variance. If quasi *F* ratios are required for the design, the computer program requests an entry of the C (Cochran's) or the S (Satterthwaite's) method for calculating quasi *F* ratios.

Output. The computer calculates the general model for the terms in the analysis. The user has the option of having the general model displayed on the monitor or skipping to the specific model for the analysis of variance in question. The program displays the specific model with appropriate factors, fixed and random. Finally, the program displays the names of the terms for mean squares for numerators and denominators of each *F* ratio and each quasi *F* ratio. The program also displays the calculations required for degrees of freedom for each *F* ratio.

Example. Suppose that an investigator wants to perform a randomized block factorial analysis of variance with subjects (blocks) a random factor, Factor A a fixed factor, and Factor B a random factor (RBF-JK). The program starts by displaying a menu that describes 20 analyses of variance. Table 1 shows the options that are displayed. The user is prompted with the words:

ENTER NUMBER OF APPROPRIATE DESIGN, THEN PRESS <RETURN>

The user selects the RBF-JK design by entering the number 5 and then depressing the carriage return key. The computer responds with the message:

```
PLEASE WAIT. THE PROGRAM IS
DETERMINING THE APPROPRIATE DESIGN.

THE PROGRAM WILL TAKE UP TO TWO
MINUTES TO PERFORM CALCULATIONS
AFTER EACH ENTRY FROM THE KEYBOARD
```

The message that appears on the screen provides an estimate of the maximum amount of time that the user will have to wait between screen displays for the analysis of

The first author is with the department of Psychology, Illinois Institute of Technology, Chicago, IL 60616. The second author is with the Department of Psychology, Northwestern State University of Louisiana, Natchitoches, LA 71457.

Table 1
Analyses of Variance That Can be Evaluated by the Program

1. CRF-J	INDEPENDENT GROUPS ONE-WAY
2. RBF-J	RANDOMIZED BLOCK ANOVAR
3. CRF-JK	INDEPENDENT GROUPS TWO-WAY
4. SPF-J.K	SPLIT PLOT FACTORIAL, ONE IND. AND ONE REP.
5. RBF-JK	RANDOMIZED BLOCK FACTORIAL, TWO FACTORS
6. CRF-JKL	INDEPENDENT GROUPS THREE-WAY
7. SPF-JK.L	SPLIT PLOT FACTORIAL, TWO IND., ONE REP.
8. SPF-J.KL	SPLIT PLOT FACTORIAL, ONE IND., TWO REP.
9. RBF-JKL	RANDOMIZED BLOCK FACTORIAL, THREE FACTORS
10. CRF-JKLM	INDEPENDENT GROUPS FOUR-WAY
11. SPF-JKL.M	SPLIT PLOT FACTORIAL, THREE IND., ONE REP.
12. SPF-JK.LM	SPLIT PLOT FACTORIAL, TWO IND., TWO REP.
13. SPF-J.KLM	SPLIT PLOT FACTORIAL, ONE IND., THREE REP.
14. RBF-JKLM	RANDOMIZED BLOCK FACTORIAL, FOUR FACTORS
15. CRF-JKLMN	INDEPENDENT GROUPS FIVE-WAY
16. SPF-JKLM.N	SPLIT PLOT FACTORIAL, FOUR IND., ONE REP.
17. SPF-JKL.MN	SPLIT PLOT FACTORIAL, THREE IND., TWO REP.
18. SPF-JK.LMN	SPLIT PLOT FACTORIAL, TWO IND., THREE REP.
19. SPF-J.KLMN	SPLIT PLOT FACTORIAL, ONE IND., FOUR REP.
20. RBF-JKLMN	RANDOMIZED BLOCK FACTORIAL, FIVE FACTORS

variance in question. This message is not repeated after it is removed from the screen.

The experimental design model equation is then shown on the screen. The program shows the following model for the present example:

```
(PI )i + (ALPHA )j + (PI ALPHA)ij  (BETA )k + (PI BETA)ik +
  (ALPHA BETA )jk + (PI ALPHA BETA )ijk +
```

The models presented by the present program are analogous to the models presented by Kirk (1982). Greek letters are printed as English words because many computer monitors cannot display greek letters. An expression such as (PI ALPHA BETA) should be read as the product of the greek letters for PI, ALPHA, and BETA. The letters that follow the words for Greek letters (i, j, etc.) should be taken as subscripts to the words for Greek letters. Note that the model printed on the monitor screen terminates with a + sign. The terminal + sign for the present model and other models can be ignored to make the model equation comparable to the equation in Kirk's book.

The program then presents expected values for mean squares for the design in question. Table 2 shows the expected values that are presented for the RBF-JK example. The table is analogous to the expected values table on page 391 of Kirk's (1982) book. Note that the subscripts in the column headed "Model" are the subscripts for successive terms in the experimental design model. The example has two factors, A and B or j and k. The columns headed by l, m, and n have no column entries because the present analysis of variance does not have factors C, D, and E.

Users are given the option of seeing the general model for the experimental design when the computer displays:

```
DO YOU WANT TO SEE THE GENERAL MODEL BEFORE
GENERATING F RATIOS FOR A SPECIFIC FIXED, MIXED,
OR RANDOM MODEL? (Y/N)
```

If the user responds with an entry of the letter Y, the computer responds with:

```
((1-p)/P)((1-q)/Q) (VAR) PI ALPHA BETA +
  n((1-q)/Q) (VAR) PI BETA +
((1-p)/P)q (VAR) PI ALPHA + nq (VAR) ALPHA +
((1-q)/Q) (VAR) PI ALPHA BETA + q (VAR) PI ALPHA +

PRESS <RETURN> TO CONTINUE
```

When the user presses the return key, the computer continues to print the terms that make up the general model. The user may have to press the return key several times before the entire model is printed on the screen. The general model is printed in a way that is analogous to the method for displaying the general model in Kirk's (1982) book. Again the notation is modified because the monitor cannot display Greek letters, subscripts, and superscripts. The letters VAR in parentheses should be read as a lowercase sigma followed by superscript 2 (the symbol for variance). The words for greek letters should be read as subscripts to the symbol for variance.

At this point the computer determines if subjects form a random factor with the question:

```
ARE SUBJECTS RANDOM OR FIXED? (R/F)
```

In the present example and in almost all studies performed by psychologists, subjects (blocks) form a random factor. The user responds by pressing the R key. Then the computer asks the question:

```
IS FACTOR A (FIRST FACTOR) RANDOM OR FIXED? (R/F)
```

Table 2
Expected Values for RBF-JK Model

	i	j	k	l	m	n
model	n	p	q	r	s	t
i	((1−n)/N)	p	q			
j	n	((1−p)/P)	q			
ij	((1−n)/N)	((1−p)/P)	q			
k	n	p	((1−q)/Q)			
ik	((1−n)/N)	p	((1−q)/Q)			
jk	n	((1−p)/P)	((1−q)/Q			
ijk	((1−n)/N)	((1−p)/P)	((1−q)/Q)			

Table 3
Specific Equations for the Seven Terms in the Experimental Design Model

p (VAR) PI BETA + pq (VAR) PI
(VAR) PI ALPHA BETA + n (VAR) ALPHA BETA + q (VAR) PI ALPHA + nq (VAR) ALPHA
(VAR) PI ALPHA BETA + q (VAR) PI ALPHA
p (VAR) PI BETA + np (VAR) BETA
p (VAR) PI BETA
(VAR) PI ALPHA BETA + n (VAR) ALPHA BETA
(VAR) PI ALPHA BETA

In our example, factor A is a fixed factor. The user responds by pressing the F key. The computer responds with:

IS FACTOR B RANDOM OR FIXED? (R/F)

The user responds by pressing the R key to indicate that Factor B is a random factor.

The computer responds by printing the specific model for the analysis of variance. That is, terms such as (1−p)/P are set to 0 or 1, depending on whether the appropriate factor is a fixed or random factor. The modifications in Kirk's (1982) notation are exactly the same as the modifications for the general model. That is, (VAR) stands for variance, and words such as ALPHA and BETA are subscripts for the (VAR) that precedes these words. Table 3 shows the seven equations that are formed for the seven terms in the model. Note that the computer requests several depressions of the return key to complete presentation of the model. Each equation is terminated by a + symbol.

Although the general and specific models may be of use to statisticians, most investigators are interested only in the *F* ratios that are determined by the specific model. The program prints the following information about *F* ratios on the monitor screen:

F BLOCKS = MSBlocks/MSBxblocks
DF NUMERATOR = DFBlocks
DF DENOMINATOR = DFBxblocks

F B = MSB/MSBxblocks
DF NUMERATOR = DFB
DF DENOMINATOR = DFBxblocks

F AB = MSAB/MSABxblocks
DF NUMERATOR = DFAB
DF DENOMINATOR = DFABxblocks

Interpretation of the above information is straightforward. For example, the *F* ratio for Factor B (F B) is obtained by dividing the mean square for Factor B (MSB) by the mean square for B by blocks (MSBxblocks). The degrees of freedom for the numerator of this *F* ratio are the degrees of freedom for Factor B (DFB), and the degrees of freedom for the denominator of this *F* ratio are the degrees of freedom for the term B by blocks (Bxblocks).

The *F* ratio for Factor A was not presented up to this point because Factor A requires a quasi *F* ratio. See Cochran (1951), Kirk (1982), and Satterthwaite (1946) for discussions of quasi *F* ratios. The computer program prints the following message on the monitor screen:

DO YOU WANT TO FORM QUASI *F* RATIOS WITH METHOD OF SATTERTHWAITE (S) OR COCHRAN (C)? (S OR C)

Let us assume that the user wants to use the method of Cochran. After a depression of the C key, the computer responds with:

QUASI F A = (MSA + MSABxblocks)/(MSAxblocks + MSAB)
DF NUMERATOR = (MSA + MSABxblocks)**2/(MSA**2/DFA + MSABxblocks**2/DFABxblocks)
DF DENOMINATOR = (MSAxblocks + MSAB)**2/ (MSAxblocks**2/DFAxblocks + MSAB**2/DFAB)

The numerator for the quasi *F* ratio for Factor A is the sum of the mean squares for Factor A and ABxblocks. The denominator for the quasi *F* ratio for Factor A is the sum of the mean squares for Axblocks and Factor AB. Note that the program uses ** to indicate raising a number to a power. The degrees of freedom for the numerator of the *F* ratio are calculated with the equation:

$$\text{df numerator} = \frac{(\text{MSA} + \text{MSABxblocks})^2}{\dfrac{\text{MSA}^2}{\text{DFA}} + \dfrac{\text{MSABxblocks}^2}{\text{DFABxblocks}}}$$

Degrees of freedom for the denominator of the *F* ratio are calculated with the equation:

$$\text{df denominator} = \frac{(\text{MSAxblocks} + \text{MSAB})^2}{\dfrac{\text{MSAxblocks}^2}{\text{DFAxblocks}} + \dfrac{\text{MSAB}^2}{\text{DFAB}}}$$

Availability. The program is written in the Microsoft version of BASIC that is used with IBM-compatible computers. The program is written so that it can easily be adapted to other versions of BASIC. A free printed listing of the program is available from Allen H. Wolach, Department of Psychology, Illinois Institute of Technology, Chicago, IL 60616. The program is also available on a disk for IBM-compatible computers by sending a blank disk with the request or $1 for a disk that we will supply.

A system with one disk drive and 128K bytes of memory is required. The disk contains two versions of the program. The first version is in a file called ANOVMODL.BAS and is in the ordinary IBM compatible interpreter version of BASIC. This program can be examined and modified by the user. The second version of the program is in a file called ANOVMODL.EXE. This version of the program is in compiled BASIC and runs faster than the interpreter version of the program. The interpreter version can take as long as 30 min between screen displays for five-factor analyses of variance. The compiled version does not require entering BASIC (sometimes called BASICA or GWBASIC) before loading the program. The user obtains the A prompt, places the disk containing the program in drive A and enters ANOVMODL <Return>.

REFERENCES

Cochran, W. G. (1951). Testing a linear relation among variances. *Biometrics*, **7**, 17-32.

Kirk, R. E. (1982). *Experimental design: Procedures for the behavioral sciences* (2nd Ed.) Belmont, CA: Brooks/Cole.

Satterthwaite, F. E. (1946). An approximate distribution of estimates of variance components. *Biometrics Bulletin*, **2**, 110-114.

(Revision accepted for publication May 17, 1987.)